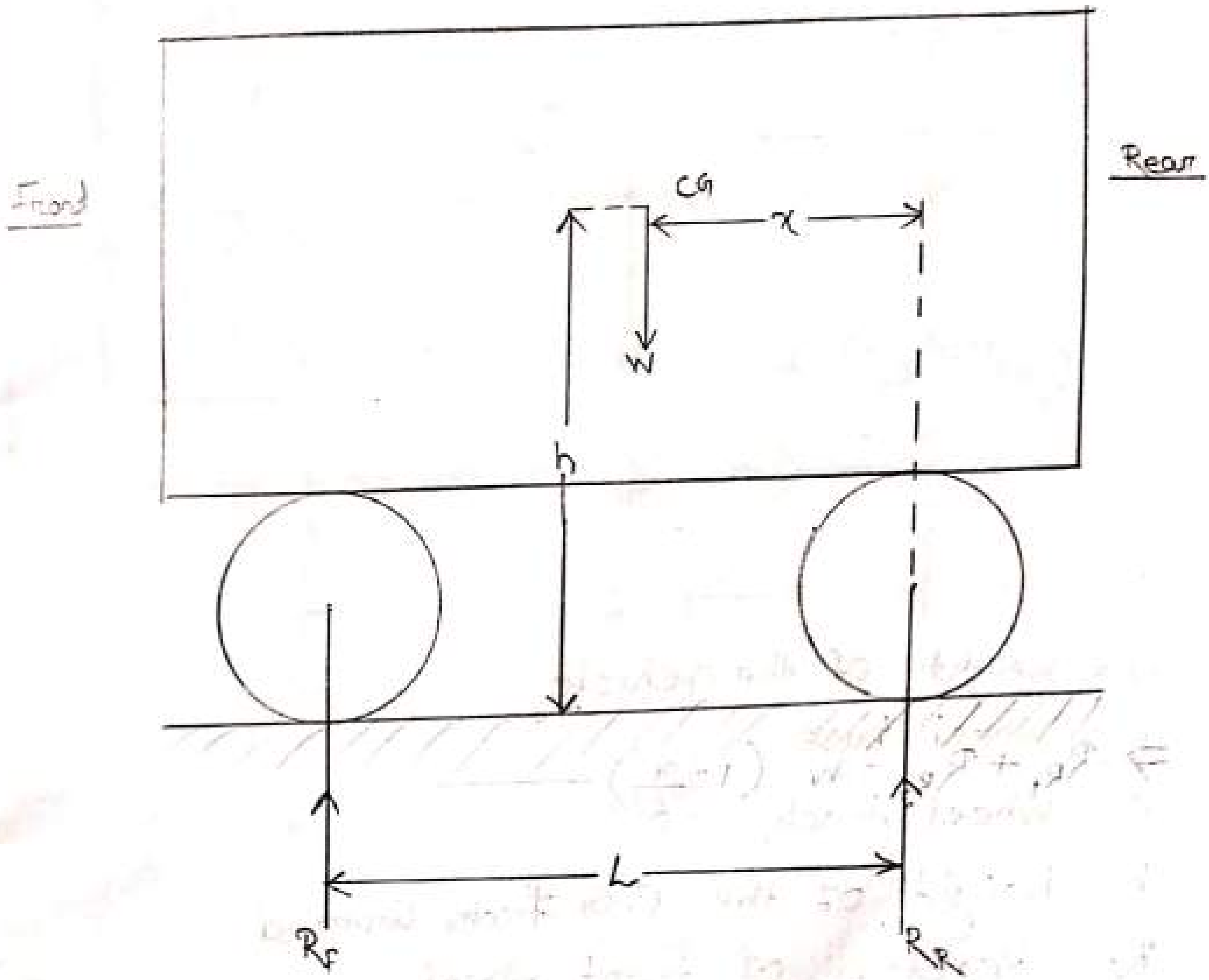
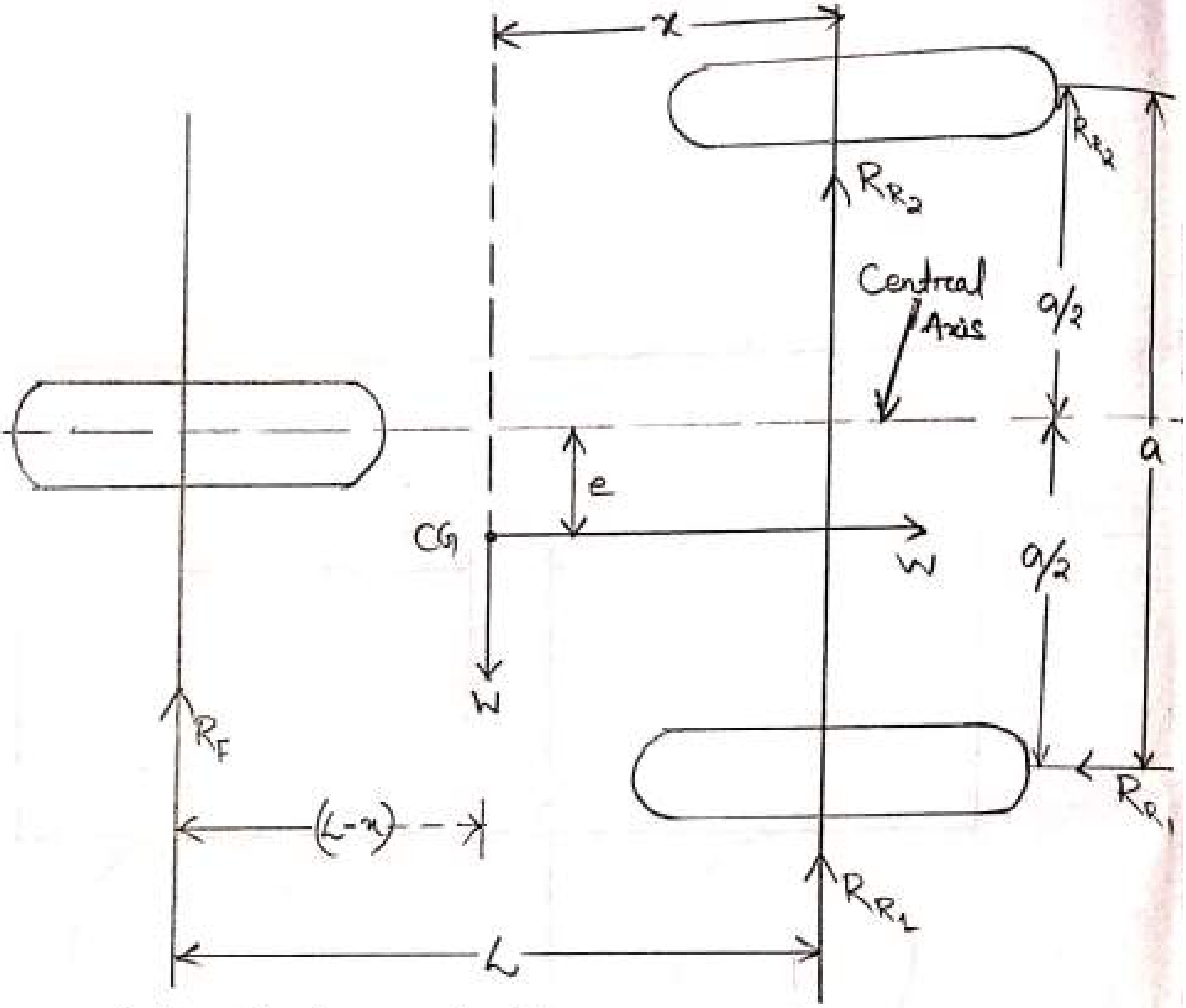
Load Distribution

(1) For three wheeler vehicle :





W = Weight of the vehicle.

L = wheel base.

a = wheel track

h = height of the C.G from Ground.

R_F = Reaction of front wheel.

R_{R_1} = Reaction of rear wheel.

R_{R_2} = Reaction of rear wheel.

e = ecentricity of C.G from Central axis.

* Moment at rear axle :-

$$+R_F \times L - Wx = 0 \Rightarrow R_F L = Wx$$

$$\Rightarrow \boxed{R_F = \frac{Wx}{L}}.$$

* Moment of Central Axle :-

$$+R_{R_1} \times \frac{a}{2} - R_{R_2} \times \frac{a}{2} - W \times e = 0$$

$$\Rightarrow \frac{a}{2}(R_{R_1} - R_{R_2}) = We$$

$$\Rightarrow \boxed{R_{R_1} - R_{R_2} = \frac{2We}{a}} \text{ ——— (1)}$$

* Moment of Front axle :-

$$-R_{R_1} \times L - R_{R_2} \times L + W(L-x) = 0$$

$$\Rightarrow W(L-x) = R_{R_1}L + R_{R_2}L = L(R_{R_1} + R_{R_2})$$

$$\Rightarrow R_{R_1} + R_{R_2} = \frac{W(L-x)}{L} = W\left(\frac{L}{L} - \frac{x}{L}\right)$$

$$\Rightarrow R_{R_1} + R_{R_2} = W\left(1 - \frac{x}{L}\right) \text{ ——— (2)}$$

Adding eq^n (1) & eq^n (2)

$$R_{R_1} - \cancel{R_{R_2}} = \frac{2We}{a}$$

$$(+) \quad R_{R_1} - R_{R_2} = W\left(1 - \frac{x}{L}\right)$$

$$\Rightarrow 2R_{R_1} = \frac{2We}{a} + W\left(1 - \frac{x}{L}\right)$$

$$= W\left[\frac{2e}{a} + 1 - \frac{x}{L}\right]$$

$$\Rightarrow R_{R_1} = \frac{W}{2}\left[\frac{2e}{a} + 1 - \frac{x}{L}\right]$$

$$\Rightarrow \boxed{R_{R_1} = \frac{W}{2}\cdot\left(1 - \frac{x}{L} + \frac{2e}{a}\right)}.$$

Putting the value of R_{R_1} in eq (1) we get.

$$R_{R_1} - R_{R_2} = \frac{2We}{a}$$

$$\Rightarrow \frac{W}{2}\left(1 - \frac{x}{L} + \frac{2e}{a}\right) - R_{R_2} = \frac{2We}{a}$$

$$\Rightarrow \frac{W}{2}\cdot\left(1 - \frac{x}{L} + \frac{2e}{a}\right) - \frac{2We}{a} = R_{R_2}$$

$$\Rightarrow R_{R_2} = \frac{W}{2}\left[1 - \frac{x}{L} + \frac{2e}{a} - \frac{4e}{a}\right]$$

$$= \frac{W}{2}\left[1 - \frac{x}{L} + \frac{2e - 4e}{a}\right]$$

$$\Rightarrow \boxed{R_{R_2} = \frac{W}{2}\left[1 - \frac{x}{L} - \frac{2e}{a}\right]}.$$

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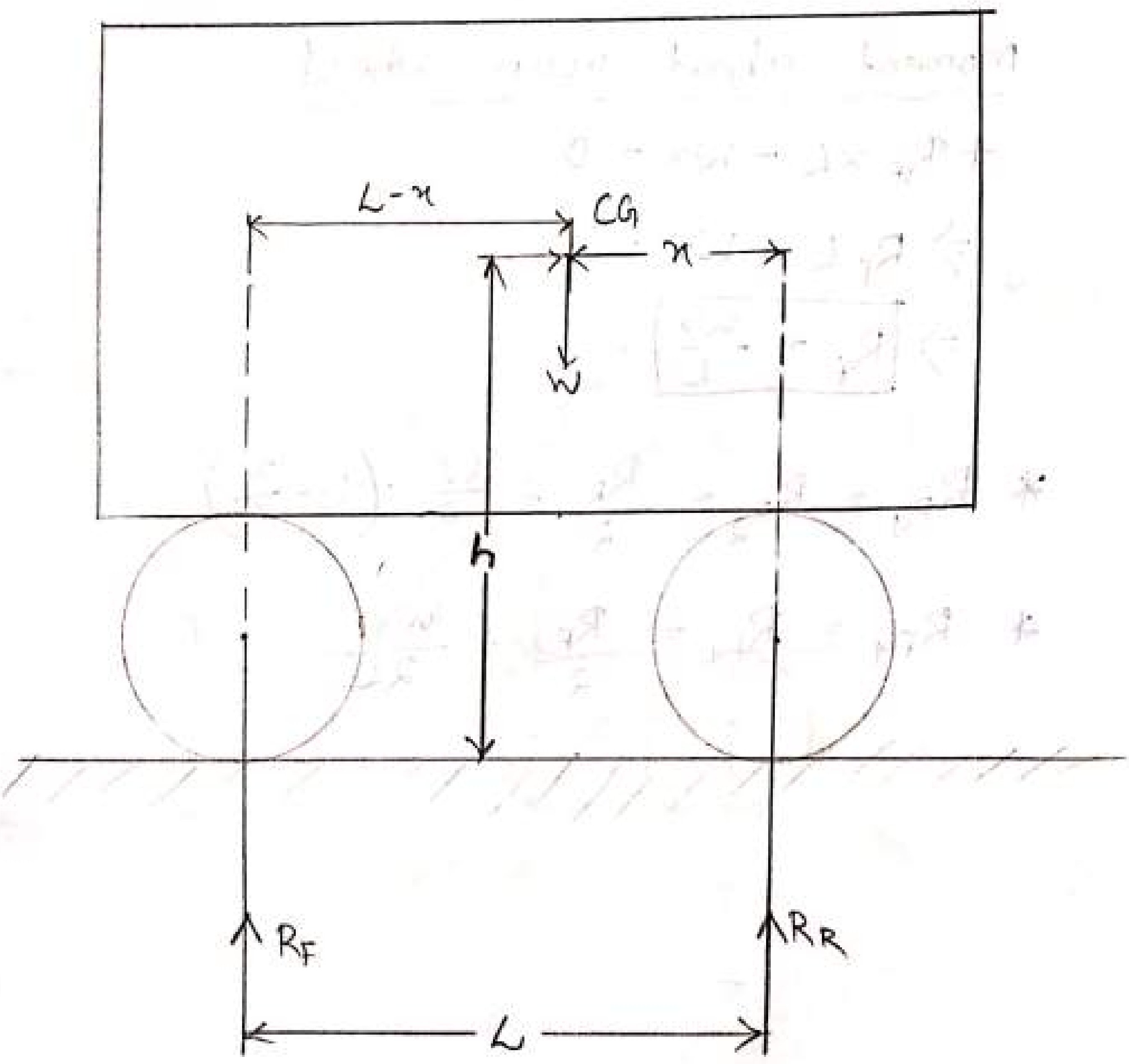
(2) Load Distribution for Four wheeler vehicle.

Front

Rear



Moment about Front axis / wheel

$$-R_R \times L + W(L-x) = 0$$

$$\Rightarrow W(L-x) = R_R \cdot L \Rightarrow R_R = \frac{W(L-x)}{L}$$

$$\Rightarrow R_R = W \cdot \left(\frac{L}{L} - \frac{x}{L}\right)$$

$$\Rightarrow \boxed{R_R = W \cdot \left(1 - \frac{x}{L}\right)}.$$

Moment about rear wheel

$$+R_F \times L - Wx = 0$$

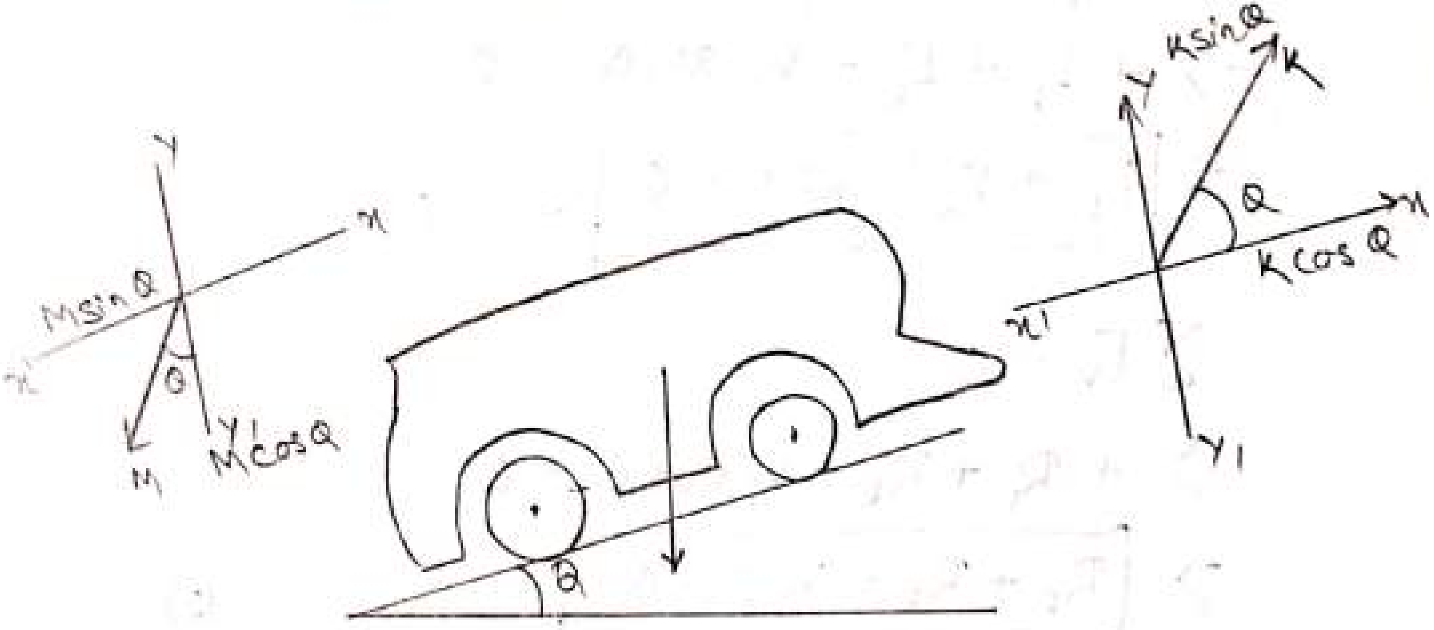
$$\Rightarrow R_F L = Wx.$$

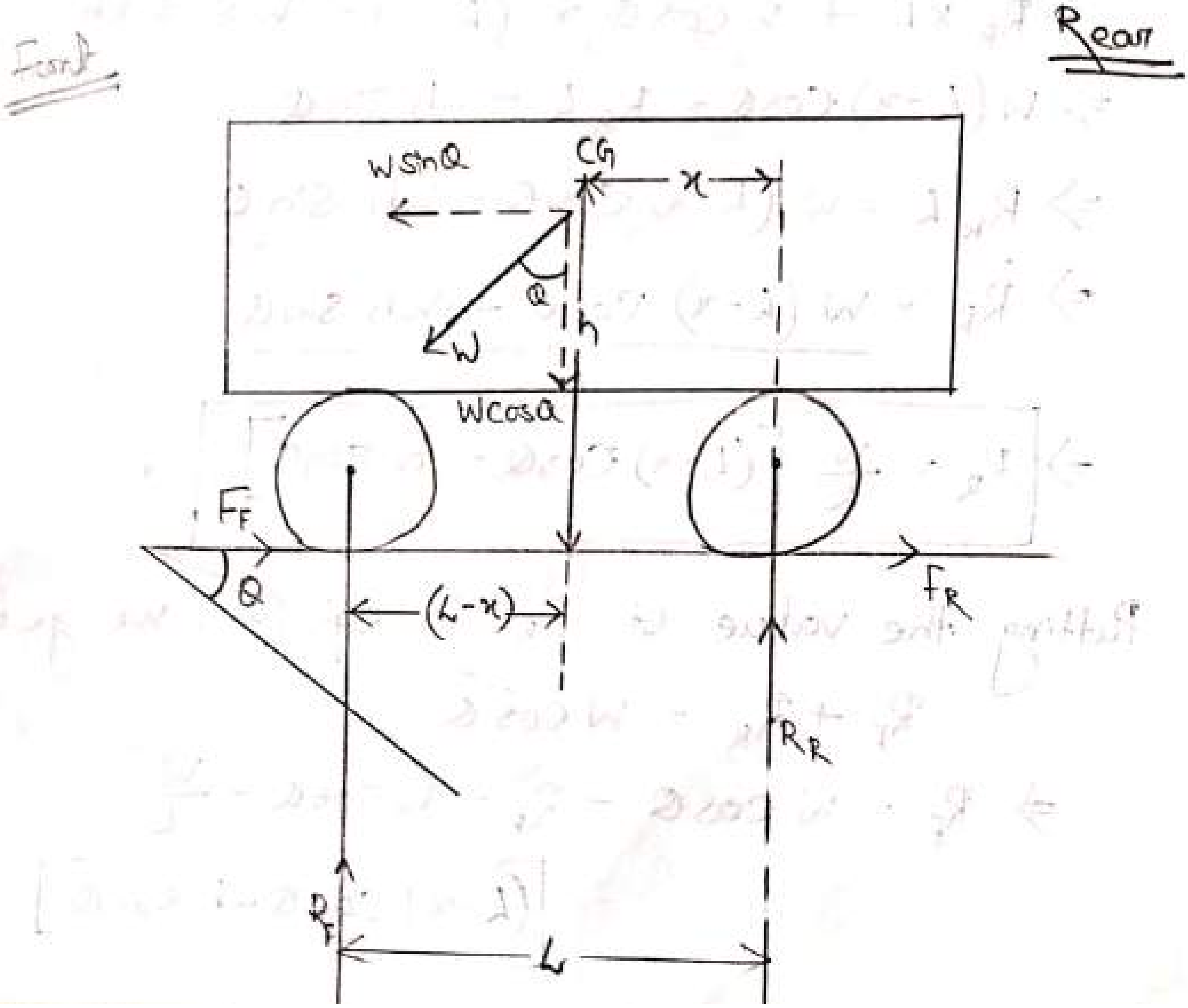
$$\Rightarrow \boxed{R_F = \frac{Wx}{L}}.$$

* $R_{R_1} = R_{R_2} = \frac{R_R}{2} = \frac{W}{2} \cdot \left(1 - \frac{x}{L}\right)$

* $R_{F_1} = R_{F_1} = \frac{R_F}{2} = \frac{Wx}{2L}$

Stability of a vehicle in a slope :-





$$\sum F_y = 0$$

$$\Rightarrow + F_F + F_R - W \sin Q = 0$$

$$\Rightarrow \boxed{F_F + F_R = W \sin Q} \quad \text{———} (1)$$

$$\sum F_v = 0$$

$$\Rightarrow + R_F + R_R - W \cos Q = 0$$

$$\Rightarrow \boxed{R_F + R_R = W \cos Q} \quad \text{———} (2)$$

Taking moment about front axle.

$$-R_R \times L + W \cos Q \times (L - x) - W \sin Q \times h = 0$$

$$\Rightarrow W(L - x) \cdot \cos Q = R_R L + Wh \sin Q$$

$$\Rightarrow R_R \cdot L = W(L - x) \cdot \cos Q - Wh \sin Q$$

$$\Rightarrow R_R = \frac{W \cdot (L - x) \cdot \cos Q - Wh \sin Q}{L}$$

$$\Rightarrow \boxed{R_R = \frac{W}{L} \left[(L - x) \cos Q - h \sin Q \right]}$$

Putting the value of R_R in eq^n (2), we get

$$R_F + R_R = W \cos Q$$

$$\Rightarrow R_F = W \cos Q - R_R = W \cos Q - \frac{W}{L} \left[(L - x) \cos Q - h \sin Q \right]$$

$$\Rightarrow R_F = \frac{W}{L}\left[L\cos\theta - (L\cos\theta - x\cos\theta - h\sin\theta)\right]$$

$$= \frac{W}{L}\left[\cancel{L\cos\theta} - \cancel{L\cos\theta} + x\cos\theta + h\sin\theta\right]$$

$$\Rightarrow \boxed{R_F = \frac{W}{L}\left[x\cos\theta + h\sin\theta\right]}.$$

Case : 1

When $\theta = \theta_s$

θ_s = Sliding angle.

$$F = \mu R$$

$$\Rightarrow (F_F + F_R) = \mu(R_F + R_R)$$

$$\Rightarrow \cancel{W}\cdot\sin\theta = \mu \cdot \times \cancel{W}\cos\theta$$

$$\Rightarrow \mu\cos\theta_s = \sin\theta_s$$

$$\Rightarrow \mu = \frac{\sin\theta_s}{\cos\theta_s}$$

$$\Rightarrow \mu = \tan\theta_s$$

$$\Rightarrow \boxed{\theta_s = \tan^{-1}\mu}.$$

Case : 2

When $\theta = \theta_o$

θ_o = Overturning angle

$$\boxed{R_R = 0}$$

$$\Rightarrow \frac{W}{L}\left[(L-x)\cos\theta - h\sin\theta\right] = 0$$

$$\Rightarrow (L-x)\cos\theta - h\sin\theta = 0$$

$$\Rightarrow (L-x)\cos\theta = h\sin\theta$$

$$\Rightarrow \frac{L-x}{h} = \frac{\sin\theta_0}{\cos\theta_0} = \tan\theta_0$$

$$\Rightarrow \boxed{\theta_0 = \tan^{-1}\left(\frac{L-x}{h}\right)}$$

Dynamic of vehicle running on a Banked Road :-

W = weight of the vehicle

L = wheel base

a = wheel track

h = height of the C.G from Ground

R_F = Reaction of front wheel.

R_{R_1} = Reaction of rear wheel.

R_{R_2} = Reaction of rear wheel.

e = ecentricity of C.G from the Central axis.

Vibration

Dt - 12/02/2020

Elements of vibration :- (Ch-1)

(1) Periodic Motion.

→ A motion which repeats itself after equal intervals of time.

(2) Time Period

→ Time taken to complete one cycle.

(3) Frequency

→ Number of cycle of unit time. (Hz)

(4) Amplitude

→ The maximum displacement of a vibrating body from it equilibrium position.

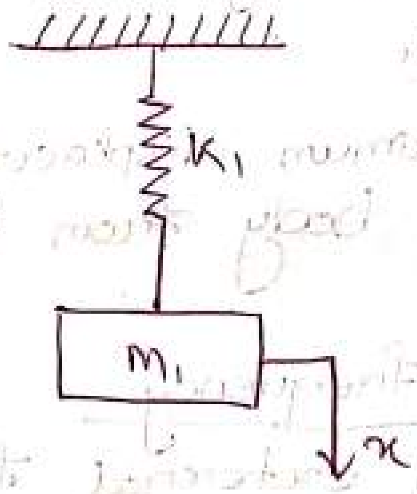
(5) Natural frequency

→ When no external force acts on the System. after giving it a initial displacement, the body vibrate. These vibration are called free vibration. There frequency is called Natural frequency. It is expressed in rad/sec or Hertz(Hz).

(6) Degree of freedom

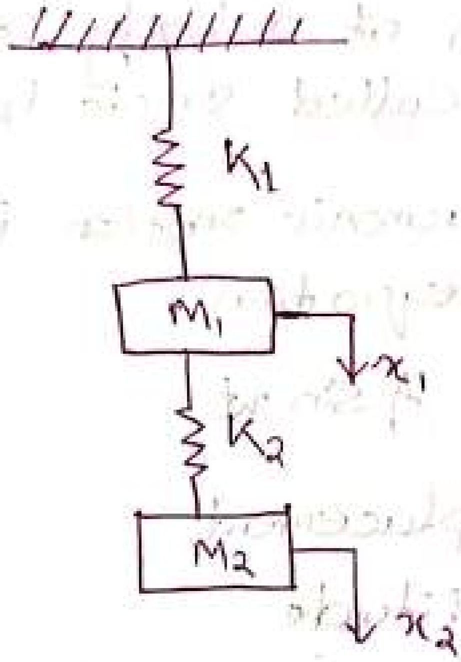
→ The minimum number of indipendent cordinant to required to specify the motion of a system insets is known as degree of freedom of the system.

(7) Single degree of freedom



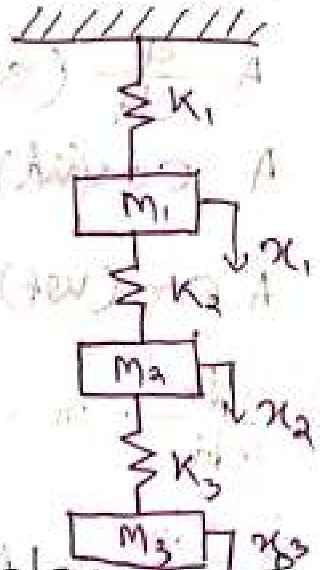
→ In single degree of freedom there is only one co-ordinate indipendent x_1 to specify the system.

(8) Two degree of freedom



→ In the system there are two indipendent co-ordinates x_1 & x_2 to specify the motion of the system.

(9) Three degree of motion



→ In three degree of freedom there are three indipendent co-ordinates x_1 & x_2 and x_3

(9) Simple Harmonic Motion

→ The motion of a body about a fixed point is called simple harmonic motion.

→ Simple harmonic motion is reprented by the equation.

$$x = A \sin wt$$

x = Displacement

A = Amplitude

t = time

w = Natural frequency.

$$\Rightarrow \frac{dx}{dt} = \frac{d}{dt}(A \sin wt)$$

$$= A \frac{d}{dt}(\sin wt)$$

$$= A \cos(wt) \times \frac{d}{dt}(wt)$$

$$= A \cos(wt) \times \frac{dt}{dt}$$

$$\Rightarrow \frac{dx}{dt} = x = A w \cos(wt)$$

$$\Rightarrow \frac{dx}{dt} = A w \cos wt$$

$$\Rightarrow \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d}{dt}(A w (\cos wt)$$

$$\Rightarrow \frac{d^2x}{dt^2} = x = A w \frac{d}{dt} \cos wt.$$

$$\frac{d}{d\theta} \sin\theta = \cos\theta$$

$$\frac{d}{d\theta} \cos\theta = -\sin\theta$$

$$\frac{d}{d\theta}(10\theta) = 10\frac{d\theta}{d\theta} = 10$$

$$\frac{d}{d\theta}(\sin 7\theta)$$

$$= \cos 7\theta \times \frac{d}{d\theta}(7\theta)$$

$$= 7 \cos 7\theta \times \frac{d\theta}{d\theta}$$

$$= 7 \cos 7\theta$$

$$= A\omega\left[-\sin(\omega t)\times\frac{d}{dt}(\omega t)\right]$$

$$= A\omega\left(-\sin\omega t\times\omega\,\frac{dt}{dt}\right)$$

$$= -A\omega\sin\omega t\times\omega$$

$$\Rightarrow \boxed{x = A\omega^2\sin\omega z}$$

$$\Rightarrow x = -\omega^2 A\sin\omega t$$

$$\Rightarrow \boxed{x = -\omega^2 x}$$

Damping :-

→ It is the resistance to the motion of a vibrating body. The vibrations associated with this resistance are known as damped vibration.

Resonance :-

→ When the frequency of external excitation is equal to the natural frequency of a vibrating body, the amplitude of vibration, becomes excessively large. This concept is known as resionance.

Parts of a vibrating System :-

→ A vibratory System basically consists of three elements.

(1) Mass

(2) Spring

(3) Damper

→ In a vibrating body there is exchange of energy from one form to another.

→ Energy store is by mass in the form of kinetic energy.

$E = \frac{1}{2} m v^2$

$E = \frac{1}{2} m (\dot{x})^2$

→ The energy is stored in the Spring in the form of potential energy.

$P = \frac{1}{2} k \cdot x^2$

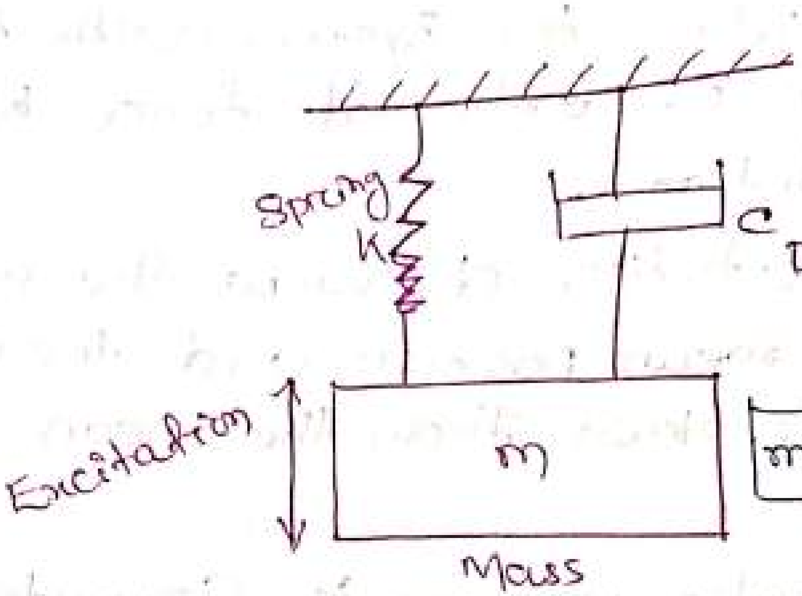
→ The energy is dicipeted in the damper in the form of heat energy. which oppose the motion of the system.

→ Energy entre the system with the application of external force known as excitation.

→ The excitation disturbs the mass from its mean position and the mass goes up & down from the mean position.

→ The kinetic energy is converted into potential energy and potential energy is converted into kinetic energy.

→ This sequence goes on repeating and the system continious to vibrate. At the same time damping force, $c\dot{x}$ acts on the mass and oppose the motion. Thus some energy is decipeted in each cycle of vibration due to damping. The free vibration die out the system remains at its static equilibrium position.



$$\boxed{m\ddot{x} + c\dot{x} + kx = 0}$$

→ Where $m\ddot{x}$ = Inertia force.

$C\dot{x}$ = Demping force

Kx = Spring force.

Methods of vibration analysis

→ There are three method:

1. Energy method.
2. Rayleigh's method.
3. Equilibrium method.

① Energy method

→ According to this method the sum of energy associated with the System is constant.

→ Mathmatically

Kinetic energy + Potential engy = Constant.

$\Rightarrow \frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2 = \text{Constant}.$

$\Rightarrow \frac{d}{dt}\left(\frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2\right) = \frac{d}{dt}(\text{const}) = 0$

$\cancel{\Rightarrow \frac{d}{dt}\frac{1}{2}m\dot{x}^2 + \frac{d}{dt}\frac{1}{2}kx^2 = 0}$

$\cancel{\Rightarrow \frac{1}{2}\left[\frac{d}{dt}m\dot{x}^2 + \frac{d}{dt}kx^2\right]}$

$\Rightarrow \frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2 = \int 0 \times dt = 0$

$\Rightarrow \frac{1}{2}\left[m\dot{x}^2 + kx^2\right] = 0$

$\Rightarrow m\dot{x}^2 + kx^2 = 0$

$\Rightarrow \frac{d}{dt} m\dot{x}^2 + \frac{d}{dt} kx^2 = \frac{d}{dt} 0$

$\Rightarrow m \frac{d}{dt}\dot{x}^2 + k\frac{d}{dt}x^2 = 0$

$\Rightarrow m \times 2\dot{x} \times \frac{d\dot{x}}{dt} + k2x \times \frac{dx}{dt} = 0$

$\Rightarrow 2m\dot{x}\ddot{x} + 2kx\dot{x} = 0$

$\Rightarrow \boxed{m\dot{x}\ddot{x} + kx = 0}$ (or)

$\Rightarrow \boxed{m\ddot{x} + kx = 0}$ —— (1).

→ If the motion is simple harmonic

$x = A \sin \omega t$

$\Rightarrow \dot{x} = A\omega \cos \omega t$

$\ddot{x} = -A\omega^2 \sin \omega t$

Putting the value eq (1) we get

$$\Rightarrow -MA\omega^2 \sin \omega t + KA \sin \omega t = 0$$

$$\Rightarrow A \sin \omega t (-M\omega^2 + K) = 0$$

$$\Rightarrow -M\omega^2 + K = 0$$

$$\Rightarrow M\omega^2 = K$$

$$\Rightarrow \omega^2 = K/m$$

$$\Rightarrow \boxed{\omega = \sqrt{K/m}} \text{ rad/sec}$$

$$\omega = 2\pi f$$

$$\Rightarrow \boxed{f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}}$$

(2) <u>Rayleigh's method</u>

→ This method is the extention of energy method. This method is based on principle that the total energy kinetic of a vibration is equal to total potential energy of the vibrating System.

Kinetic energy = potential energy

$$K \cdot E = \frac{1}{2} m\dot{x}^2$$

$$= \frac{1}{2} m A^2 \omega^2 \cos^2(\omega t)$$

$$\Rightarrow (K \cdot E)_{max} = \frac{1}{2} m A^2 \omega^2$$

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$$P.E = \frac{1}{2}kx^2$$

$$= \frac{1}{2}k(A\sin\omega t)^2$$

$$= \frac{1}{2}kA^2 \sin^2(\omega t)$$

$$\Rightarrow (P.E)_{max} = \frac{1}{2}kA^2$$

$$(K.E)_{max} = (P.E)_{max}$$

$$\Rightarrow \cancel{\frac{1}{2}} m\cancel{A^2}\omega^2 = \cancel{\frac{1}{2}} k\cancel{A^2}$$

$$\Rightarrow m\omega^2 = k$$

$$\boxed{\Rightarrow \omega^2 = \frac{k}{m}}$$

$$\Rightarrow \boxed{\omega = \sqrt{\frac{k}{m}}}$$

$$w = 2\pi f$$

$$\Rightarrow f = \frac{\omega}{2\pi}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

(3) Equilibrium Method

→ According to this method the algebric sum of the forces and moments acting on the system must be 0.

→ If the external force acting on the System = F

→ Inertia force = $m\ddot{x}$

Damping force = $C\dot{x}$

Spring force = Kx

Then equation of motion can be written as →

$$\boxed{M\ddot{x} + C\dot{x} + Kx = F}$$

Dt - 14/02/2020

Free Damped vibration

Damping :-

Damping is the resistance by a body to the motion of a vibrating system.

Types of Damping

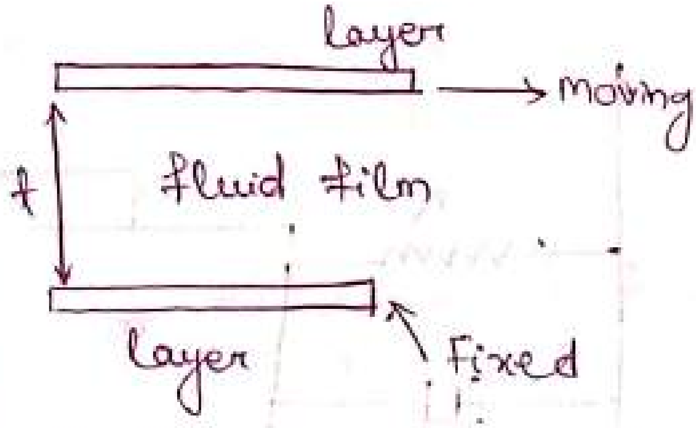
It is the four types.

(1). Viscous damping
(2) Coulomb damping.
(3) Structural damping
(4) Known linear damping.

(1) Viscous Damping

→ Viscocusity is the property of a fluid by vetdue of which it offers resistance to the motion of one layer. over adjacent one.

→ When the system is allowed to vibrate in a viscous midium, the damping is called viscous damping.



→ Let us Consider

A = Area of the Plate

t = Thickness of the fluid field

V = velocity

μ = Co-efficient of viscousity

F = Total force.

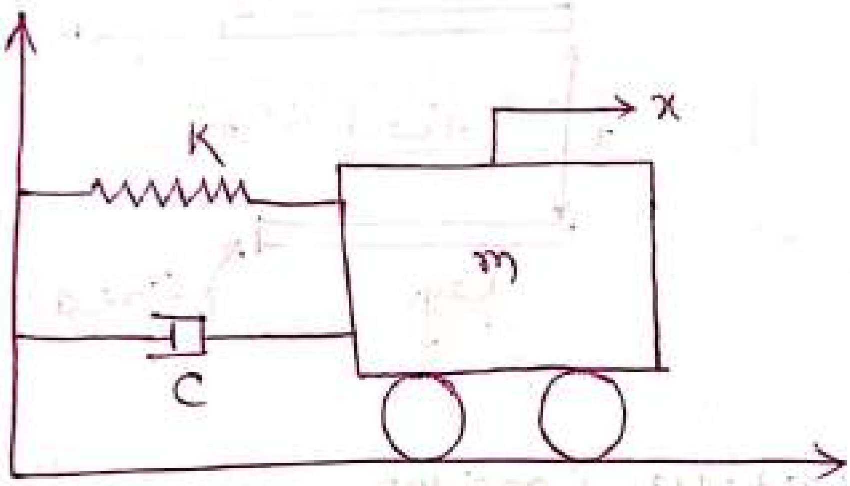
$$\boxed{F = \frac{\mu A \dot{x}}{t}}$$

If

$C = \frac{\mu A}{t}$ = viscous damping co-efficient

$F = \frac{\mu A}{t} \times \dot{x} = C\dot{x}$.

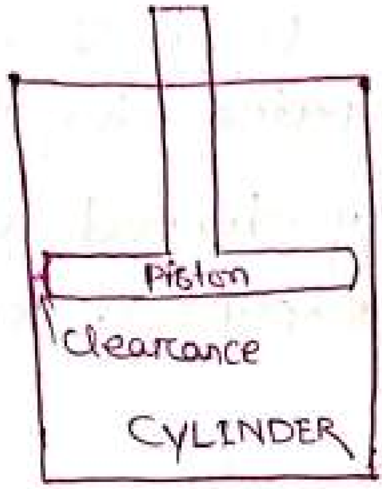
$\Rightarrow \boxed{F = C\dot{x}}$.



Eq^n of the motion

$$\boxed{m\ddot{x} + C\dot{x} + Kx = 0}.$$

K = Spring
C = Damper
m = mass
x = Displacement.



→ The ~~se~~ main components of viscous damper are → Cylinder, piston and viscous fluid.

→ There is a clearance between cylinder wall & piston.

→ More the clearance, more will be the velocity of the piston in the viscous fluid and it will offer less value of viscous damping co-efficient.

* Energy Dessipation in viscous damping :

→ For a vibratory body some amount of energy is dessipated due to damping.

→ For a viscously damped system the force 'F' is expressed as –

$$F = C\dot{x}$$

energy = work × time
work = force × displacement

$$\Rightarrow F = C\frac{dx}{dt}$$

$$\text{Work done} = dW = F \times dx = C\left(\frac{dx}{dt}\right)dx$$

rate of change of work –

$$\frac{dW}{dt} = \frac{F\,dx}{dt} = F\left(\frac{dx}{dt}\right)$$

$$\omega = 2\pi f$$

$$\Rightarrow \boxed{t = \frac{2\pi}{\omega}}$$

rate of change of work per cycle.

i.e

ΔE = Energy desipeted

$$\Delta E = \int_0^t \left(\frac{dW}{dt}\right)dt$$

$$\Rightarrow \Delta E = \int_0^t F\left(\frac{dx}{dt}\right)dt$$

$$= \int_0^t C \left(\frac{dx}{dt} \times \frac{dx}{dt}\right) dt$$

$$= \int_0^t C \left(\frac{dx}{dt}\right)^2 dt \quad \text{——— (1)}$$

Let the motion is simple harmonic

$$x = A \sin \omega t$$

$$\Rightarrow \dot{x} = \frac{dx}{dt} = \frac{d}{dt}(A \sin \omega t)$$

$$\Rightarrow \frac{dx}{dt} = A\,\omega \cos \omega t$$

$$\Rightarrow \left(\frac{dx}{dt}\right)^2 = A^2 \omega^2 \cos^2 \omega t$$

$$\boxed{\begin{aligned} \cos 2\theta &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta \\ &= \cos^2\theta - \sin^2\theta \\ &= \frac{1-\tan^2\theta}{1+\tan^2\theta} \\ \Rightarrow 2\cos^2\theta &= 1 + \cos 2\theta \end{aligned}}$$

$$\boxed{\Rightarrow \cos^2\theta = \frac{\cos 2\theta + 1}{2}}$$

→ Now putting the value in eq (1) we get

$$\Delta E = \int_0^t C A^2 \omega^2 \cos^2(\omega t)\, dt$$

$$\boxed{\cos^2(\omega t) = \frac{\cos(2\omega t)+1}{2}}$$

$$\Delta E = C A^2 \omega^2 \int_0^t \cos^2(\omega t)\, dt$$

$$= C A^2 \omega^2 \int_0^t \frac{\cos(2\omega t) + 1}{2}\, dt$$

$$= \frac{C A^2 \omega^2}{2} \int_0^t \cos(2\omega t) + 1\, dt$$

$$= \frac{C A^2 \omega^2}{2} \left[\int_0^t \cos(2\omega t)\, dt + \int_0^t 1\, dt\right]$$

$$= \frac{C A^2 \omega^2}{2} \left\{\left[\frac{\sin(2\omega t)}{2\omega}\right]_0^{\frac{2\pi}{\omega}} + \left[t\right]_0^{2\pi/\omega}\right\}$$

$$= \frac{CA^2\omega^2}{2} \left\{ \left[\frac{\sin(2\omega \times \frac{2\pi}{\omega}) - \sin(2\omega \times 0)}{2\omega} \right] + \left[\frac{2\pi}{\omega} - 0 \right] \right\}$$

$$= \frac{CA^2\omega^2}{2} \left\{ \frac{\sin 4\pi - \sin 0}{2\omega} + \frac{2\pi}{\omega} \right\}$$

$$= \frac{CA^2\omega^2}{2} \left(\frac{0-0}{2\omega} + \frac{2\pi}{\omega} \right)$$

$$= \frac{CA^2\omega^2}{2} \times \frac{2\pi}{\omega} = CA^2\omega\pi$$

$$\Rightarrow \boxed{\Delta E = \pi C\omega A^2}.$$

→ Energy dessipation per cycle ΔE is directly propotional to the square of the amplitude of the motion.

$$\boxed{\Delta E \propto A^2}.$$

→ The total energy of a vibrating system can be either maximum of its potential energy or kinetic energy.

$$E = (K.E)_{max} \quad \text{——— (1)}$$

$$K.E = \frac{1}{2} m\dot{x}^2$$

$= \frac{1}{2} m (A\omega \cos \omega t)^2 = \frac{1}{2} m A^2 \omega^2 \cos^2(\omega t)$

$(K.E)_{max} = \frac{1}{2} m A^2 \omega^2$.

Now putting the value eq(1) we get

$\Rightarrow \boxed{E = \frac{1}{2} m A^2 \omega^2}$.

Specific damping Capacity :-

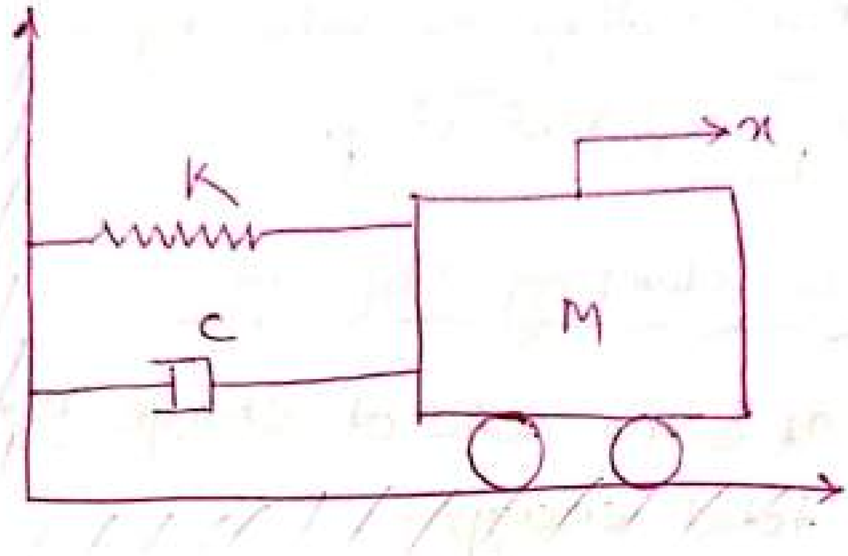
β = It is the ratio of change in energy to total energy .

$= \frac{\Delta E}{E}$

$= \frac{\pi c \omega A^2}{\frac{1}{2} m A^2 \omega^2}$

$= \boxed{\beta = \frac{2\pi c}{m\omega}}$ (Which is a constant)

Differential equation of damped free vibration :-



→ The various forces in this systems are :-

(i) Acelerating force = $m\ddot{x}$

(ii) Damping force = $c\dot{x}$

(iii) Spring force = Kx

Thus the equation of motion can be written as → $\boxed{m\ddot{x} + c\dot{x} + Kx = 0}$ — (1)

This is called characteristic equation of system.

$m\ddot{x} + c\dot{x} + kx = 0$

$$\Rightarrow m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0 \quad \text{—— (1)}$$

The solution of

$$x = A_1 e^{u_1 t} + A_2 e^{u_2 t}$$

Where u_1 & u_2 are called the roots.

A_1 and A_2 is arbitrary Constant.

Assuming in a solution of the form

$$x = e^{ut}$$

$$\Rightarrow \frac{dx}{dt} = \frac{d}{dt}e^{ut} = e^{ut} \times \frac{d}{dt}(ut)$$

$$\Rightarrow \frac{dx}{dt} = e^{ut} \times u = ue^{ut}$$

$$\Rightarrow \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d}{dt}(ue^{ut})$$

$$\Rightarrow \frac{d^2x}{dt^2} = u\left(\frac{d}{dt}e^{ut}\right) = u \times ue^{ut}$$

$$\Rightarrow \frac{d^2x}{dt^2} = u^2 e^{ut}$$

$$\boxed{\begin{aligned} &\frac{d}{d\theta}e^{\theta} = e^{\theta} \\ &\frac{d}{d\theta}e^{(5\theta)} \\ &= e^{5\theta} \times \frac{d}{d\theta}5\theta \\ &= e^{5\theta} \times 5 = 5e^{5\theta} \end{aligned}}$$

Now putting the value in eq. (1) we get

$$M\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

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$m \times u^2 e^{ut} + c \times u e^{ut} + \cancel{k} \; k \times e^{ut} = 0$

Now dividing me^{ut} in its term we get,

$$\frac{\cancel{m} u^2 \cancel{e^{ut}}}{\cancel{m} \cancel{e^{ut}}} + \frac{c u \cancel{e^{ut}}}{m \cancel{e^{ut}}} + \frac{k \cancel{e^{ut}}}{m \cancel{e^{ut}}}$$

$$\Rightarrow u^2 + \left(\frac{c}{m}\right) u + \frac{k}{m} = 0$$

$$\boxed{\begin{array}{l} ax^2 + bx + c = 0 \\ \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{array}}$$

Sol^n,

$$u = \frac{-\frac{c}{m} \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4 \cdot 1 \cdot \frac{k}{m}}}{2 \times 1}$$

$$= -\frac{c}{2m} \pm \sqrt{\frac{1}{4}\left(\frac{c}{m}\right)^2 - \frac{4k}{4m}}$$

$$= -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$u_2 / u_1 = -\frac{c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}.$$

$$u_1 / u_2 = -\frac{c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}.$$

Now putting the value of root we sol^n we get.

$$x = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t}.$$

eq^n (A)

$$\boxed{x = A_1 e^{\left[-c/2m + \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}\right]t} + A_2 e^{\left[-c/2m - \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}\right]t}}$$

Dt - 17/02/2020.

Critical damping constant (C_c)

→ Critical damping defined as C_c the value of damping co-efficient 'c' for which the mathmatically term:

$\left(\frac{c}{2m}\right)^2 - \frac{k}{m}$ in the eq^n No (A) is equal to zero

$$\left(\frac{c}{2m}\right)^2 - \frac{k}{m} = 0 \quad \text{------} \quad (2)$$

$$\Rightarrow \left(\frac{C_c}{2m}\right)^2 - \frac{k}{m} = 0$$

$$\Rightarrow \left(\frac{C_c}{2m}\right)^2 = \frac{K}{m}$$

$$\Rightarrow \frac{C_c}{2m} = \sqrt{\frac{K}{m}}$$

$$\Rightarrow \boxed{C_c = 2m\sqrt{\frac{K}{m}} = 2\sqrt{Km}}$$

$$\boxed{\begin{aligned} m\sqrt{\frac{K}{m}} &= m\frac{\sqrt{K}}{\sqrt{m}} \\ &= \frac{\sqrt{m}\times\sqrt{m}\times\sqrt{K}}{\sqrt{m}} \\ &= \sqrt{m}\times\sqrt{K} \end{aligned}}$$

$$C_c = 2m\sqrt{\frac{K}{m}} = 2m\omega$$

$$\Rightarrow C_c = 2m\omega$$

$$\Rightarrow \boxed{\frac{C_c}{2m} = \omega}$$

Damping Ratio

→ The ratio of 'c' to 'C_c' is known as damping Ratio.

ε :- c to C_c

$$\boxed{\varepsilon = \frac{c}{C_c}}$$

$$* \quad \frac{c}{2m} = \frac{c}{2m}\times\frac{C_c}{C_c} = \frac{c}{C_c}\times\frac{C_c}{2m} = \varepsilon\omega$$

$$\boxed{\begin{aligned} &\sqrt{\frac{K}{m}} = \omega \\ &\Rightarrow \frac{K}{m} = \omega^2 \end{aligned}}$$

eq^n (A) becomes

$$x = A_1 e^{\left[-\frac{c}{2m}+\sqrt{\left(\frac{c}{2m}\right)^2-\frac{K}{m}}\right]t} + A_2 e^{\left[-\frac{c}{2m}-\sqrt{\left(\frac{c}{2m}\right)^2-\frac{K}{m}}\right]t}$$

$$= A_1 e^{\left[-\varepsilon\omega+\sqrt{(\varepsilon\omega)^2-\omega^2}\right]t} + A_2 e^{\left[-\varepsilon\omega-\sqrt{(\varepsilon\omega)^2-\omega^2}\right]t}$$

$$= A_1 e^{\left(-\varepsilon\omega + \sqrt{\varepsilon^2\omega^2 - \omega^2}\right)t} + A_2 e^{\left(-\varepsilon\omega - \sqrt{\varepsilon^2\omega^2 - \omega^2}\right)t}$$

$$= A_1 e^{\left[-\varepsilon\omega + \sqrt{\omega^2(\varepsilon^2 - 1)}\right]t} + A_2 e^{\left[-\varepsilon\omega - \sqrt{\omega^2(\varepsilon^2-1)}\right]t}$$

$$= A_1 e^{\left(-\varepsilon\omega + \omega\sqrt{\varepsilon^2 - 1}\right)t} + A_2 e^{\left(-\varepsilon\omega - \omega\sqrt{\varepsilon^2 - 1}\right)t}$$

Eqn (B) $\Rightarrow$

$$\boxed{x = A_1 e^{\left(-\varepsilon \pm \sqrt{\varepsilon^2 - 1}\right)\omega t} + A_2 e^{\left(-\varepsilon - \sqrt{\varepsilon^2-1}\right)\omega t}}$$

→ Depending upon the value of damping Ratio 'ε', the damped systems are divided into three catagories.

(1) Over Damped System. $(\varepsilon > 1)$

(2) Critical Damped System $(\varepsilon = 1)$

(3) Under Damped System $(\varepsilon < 1)$

(1) Over Damped System

→ In this system ε is greater than 1. the motion is called aperiodic.

$$x = A_1 e^{\left(-\varepsilon \pm \sqrt{\varepsilon^2-1}\right)\omega t} + A_2 e^{\left(-\varepsilon - \sqrt{\varepsilon^2-1}\right)\omega t}$$

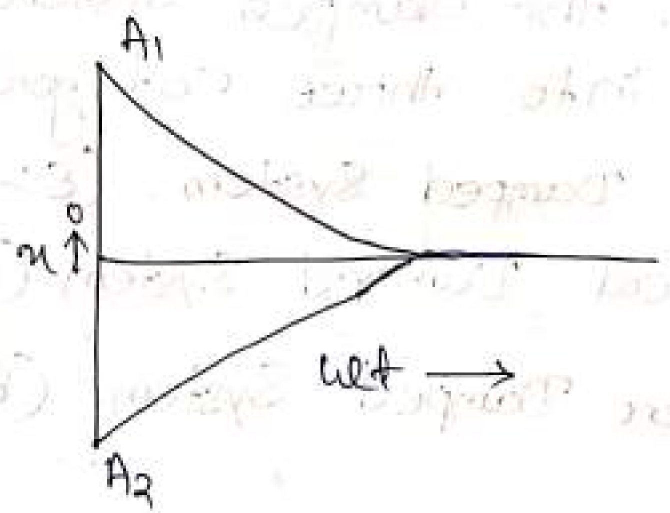
When $t = 0$

$$x = A_1 e^{(-\varepsilon + \sqrt{\varepsilon^2 - 1})\omega_n 0} + A_2 e^{(-\varepsilon - \sqrt{\varepsilon^2 - 1})\omega_n 0}$$

$$\Rightarrow x = A_1 e^0 + A_2 e^0$$

$$\boxed{x = A_1 + A_2}$$

The values of A_1 & A_2 are (+ve).



$$\frac{de^{\theta}}{d\theta} = e^{\theta} \times \frac{d\theta}{d\theta}$$

$$\frac{de^{7\theta}}{d\theta} = e^{7\theta} \frac{d7\theta}{d\theta}$$

$$= 7e^{7\theta}$$

→ The values of displacement x goes on decreasing with time. The System is non vibratory in nature.

→ The System Comes to equilibrium in an exponential manner.

$$x = A_1 e^{(-\varepsilon + \sqrt{\varepsilon^2-1})\omega t} + A_2 e^{(-\varepsilon - \sqrt{\varepsilon^2-1})\omega t}$$

$$\Rightarrow \frac{dx}{dt} = \frac{d}{dt} A_1 e^{(-\varepsilon + \sqrt{\varepsilon^2-1})\omega t} + A_2 e^{(-\varepsilon - \sqrt{\varepsilon^2-1})\omega t}$$

$$\Rightarrow \frac{dx}{dt} A_1 \times e^{(-\varepsilon + \sqrt{\varepsilon^2-1})\omega t} \times \frac{d}{dt} A_2 \times e^{(-\varepsilon - \sqrt{\varepsilon^2-1})\omega t}$$

$$\boxed{\begin{aligned} &\frac{d}{dQ} e^{\sin Q} \\ &= e^{\sin Q} \times \frac{d}{dQ} \sin Q \\ &= \cos Q \times e^{\sin Q} \end{aligned}}$$

$$\frac{dx}{dt} = A_1 (-\varepsilon + \sqrt{\varepsilon^2-1})\omega \cdot e^{(-\varepsilon + \sqrt{\varepsilon^2-1})\omega t} + A_2 (-\varepsilon - \sqrt{\varepsilon^2-1})\omega \cdot e^{(-\varepsilon - \sqrt{\varepsilon^2-1})\omega t}$$

$$\frac{dx}{dt} = A_1 \left[e^{(-\varepsilon + \sqrt{\varepsilon^2-1})\omega t} \times \frac{d}{dt} (-\varepsilon + \sqrt{\varepsilon^2-1})\omega t \right.$$

$$+ A_2 e^{(-\varepsilon - \sqrt{\varepsilon^2-1})\omega t} \times \frac{d}{dt} (-\varepsilon - \sqrt{\varepsilon^2-1})\omega t$$

$$= A_1 \left[e^{(-\varepsilon + \sqrt{\varepsilon^2-1})\omega t} \times (-\varepsilon + \sqrt{\varepsilon^2-1})\omega \cdot \frac{dt}{dt} \right]$$

$$+ A_2 \left[e^{(-\varepsilon - \sqrt{-\varepsilon^2-1})\omega t} \times (-\varepsilon - \sqrt{\varepsilon^2-1})\omega \cdot \frac{dt}{dt} \right]$$

$$\boxed{\begin{aligned} \frac{dx}{dt} = {} & A_1 (-\varepsilon + \sqrt{(\varepsilon^2-1)})\omega \cdot e^{(-\varepsilon + \sqrt{\varepsilon^2-1})\omega t} \\ & + A_2 (-\varepsilon - \sqrt{\varepsilon^2-1})\omega \cdot e^{(-\varepsilon - \sqrt{\varepsilon^2-1})\omega t} \end{aligned}}$$

Initial Condition

At $t = 0$

$$x_0 = A_1 + A_2 \quad \text{--- (1)}$$

$t = 0$, $\dot{x}_0 = A_1(-\varepsilon + \sqrt{\varepsilon^2 - 1})\omega$

$$+ A_2(-\varepsilon - \sqrt{\varepsilon^2 - 1})\omega \quad \text{--- (2)}$$

From the above eqn A_1 and A_2 can be calculated.

(2) Critical Damped System :-

→ In this case $\varepsilon = 1$.

→ The condition is :-

$$\left(\frac{c}{2m}\right)^2 - \frac{k}{m} = 0$$

$$\Rightarrow \left(\frac{c}{2m}\right)^2 = \frac{k}{m}$$

$$\Rightarrow \frac{c}{2m} = \sqrt{\frac{k}{m}}$$

* The two roots u_1 & u_2 equal to each other.

$$U = u_1 = u_2$$

$$\Rightarrow u = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$= -\frac{c}{2m} \pm \sqrt{\frac{k}{m} - \frac{k}{m}}$$

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$$\Rightarrow u = u_1 = u_2 = -\frac{c}{2m} = -\varepsilon \omega \quad (\because \varepsilon = 1)$$

$$\Rightarrow \boxed{u_1 = u_2 = -\omega}$$

The appropriate solution of the eqn

$$u^2 + \frac{c}{m} u + \frac{k}{m} = 0$$

$$x = A_1 e^{-\omega t} + A_2 t e^{-\omega t} = (A_1 + A_2 t) e^{-\omega t}$$

~~Applying initial condition~~

$$\boxed{\begin{aligned} &\frac{d}{d\theta}[f(\theta) \times g(\theta)] \\ &= g(\theta)\frac{d}{d\theta} f(\theta) \\ &\quad + f(\theta)\frac{d}{d\theta} g(\theta) \end{aligned}}$$

$$\cancel{\frac{d}{dt}\left(A_1 e^{-\omega t} + A_2 t e^{-\omega t}\right)}$$

$$\dot{x} = \frac{d}{dt} A_1 e^{-\omega t} + \frac{d}{dt} A_2 t e^{-\omega t}$$

$$= A_1 e^{-\omega t} \times \frac{d}{dt}(-\omega t) + A_2 \left[e^{-\omega t} \cdot \frac{dt}{dt} + t \cdot \frac{d}{dt} e^{-\omega t} \right]$$

$$= -A_1 \omega e^{-\omega t} + A_2 \left[e^{-\omega t} + t \cdot e^{\omega t} \cdot \frac{d}{dt}(-\omega t) \right]$$

$$= -A_1 \omega e^{-\omega t} + A_2 \left[e^{-\omega t} - \omega t e^{-\omega t} \right]$$

$$= -A_1 \omega e^{-\omega t} + A_2 e^{-\omega t} \cdot \left[1 - \omega t \right]$$

$$\dot{x} = e^{-\omega t} \cdot \left[-A_1 \omega + A_2 (1 - \omega t) \right]$$

At, $t = 0$

$$x_0 = A_1$$

$$t = 0, \quad \dot{x}_0 = e^{0} \left[-A_1 \omega + A_2 \right] = A_2 - A_1 \omega$$

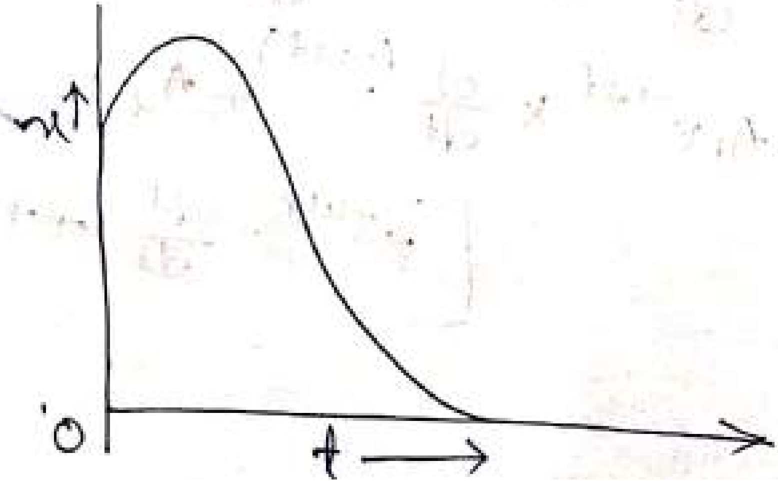
$$= A_2 - x_0 \omega$$

$$\Rightarrow \boxed{A_2 = \dot{x}_0 + x_0 \omega}.$$

Now the solution becomes

$$x = (A_1 + A_2 t) e^{-\omega t}$$

$$\boxed{x = \left[x_0 + (\dot{x}_0 + x_0 \omega) t \right] e^{-\omega t}}.$$



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The value of x decreases as t increases at finally becomes zero as t tens to α (infinity). This also an aperiodic motion.

(3) Under Damped System

$\Rightarrow \varepsilon < 1$ (In this case)

$$u_1 = \left(-\varepsilon + \sqrt{\varepsilon^2-1}\right)\omega$$

$$u_2 = \left(-\varepsilon - \sqrt{\varepsilon^2-1}\right)\omega$$

$$\Rightarrow u_1 = \left[-\varepsilon + i\sqrt{1-\varepsilon^2}\right]\omega$$

$$u_2 = \left[-\varepsilon - i\sqrt{1-\varepsilon^2}\right]\omega$$

$(\because x = A_1 e^{u_1 t} + A_2 e^{u_2 t})$

$$x = A_1 e^{\left[-\varepsilon + i\sqrt{1-\varepsilon^2}\right]\omega t} + A_2 e^{\left[-\varepsilon - i\sqrt{1-\varepsilon^2}\right]\omega t}$$

$$= A_1 e^{\left(-\varepsilon\omega t + i\omega t\sqrt{1-\varepsilon^2}\right)} + A_2 e^{\left(-\varepsilon\omega t - i\omega t\sqrt{1-\varepsilon^2}\right)}$$

$$= A_1 e^{-\varepsilon\omega t} \cdot e^{i\omega t\sqrt{1-\varepsilon^2}} + A_2 e^{-\varepsilon\omega t} \cdot e^{-i\omega t\sqrt{1-\varepsilon^2}}$$

$\sqrt{25} = 5 \times 5$

$\sqrt{-25} = -5 \times -5 = 25$ ✗

$= \sqrt{25 \times (-1)}$

$= \sqrt{25} \times \sqrt{-1}$

$= 5\sqrt{-1}$

$= 5i$

$i = \sqrt{-1}$ Imaginary number

$$\boxed{e^{x+y} = e^x \cdot e^y}$$

$$= e^{-\varepsilon\omega t}\left[A_1 e^{i\omega t\sqrt{1-\varepsilon^2}} + A_2 e^{-i\omega t\sqrt{1-\varepsilon^2}}\right]$$

we know that

$$e^{ix} = \cos x + i\sin x$$

$$e^{-ix} = \cos x - i\sin x$$

~~we know that~~

$$\Rightarrow x = e^{-\varepsilon\omega t}\left[A_1 \times \left\{\cos(\omega t\sqrt{1-\varepsilon^2}) + i\sin(\omega t\sqrt{1-\varepsilon^2})\right\}\right.$$

$$\left. + A_2 \times \left[\cos(\omega t\sqrt{1-\varepsilon^2}) - i\sin(\omega t\sqrt{1-\varepsilon^2})\right]\right]$$

$$x = e^{-\varepsilon\omega t}\left[A_1\cos\omega t\sqrt{1-\varepsilon^2} + A_1 i\sin\omega t\sqrt{1-\varepsilon^2}\right.$$

$$\left. + A_2\cos(\omega t\sqrt{1-\varepsilon^2}) - A_2 i\sin\omega t\sqrt{1-\varepsilon^2}\right]$$

$$= e^{-\varepsilon\omega t}\left[\cos\omega t\sqrt{1-\varepsilon^2}\,(A_1+A_2) + i\sin(\omega t\sqrt{1-\varepsilon^2})(A_1-A_2)\right]$$

$$x = e^{-\varepsilon\omega t}\left[C_1\cos\omega t\sqrt{1-\varepsilon^2} + C_2 i\sin\omega t\sqrt{1-\varepsilon^2}\right]$$

Where $C_1 = A_1 + A_2$

$C_2 = A_1 - A_2$

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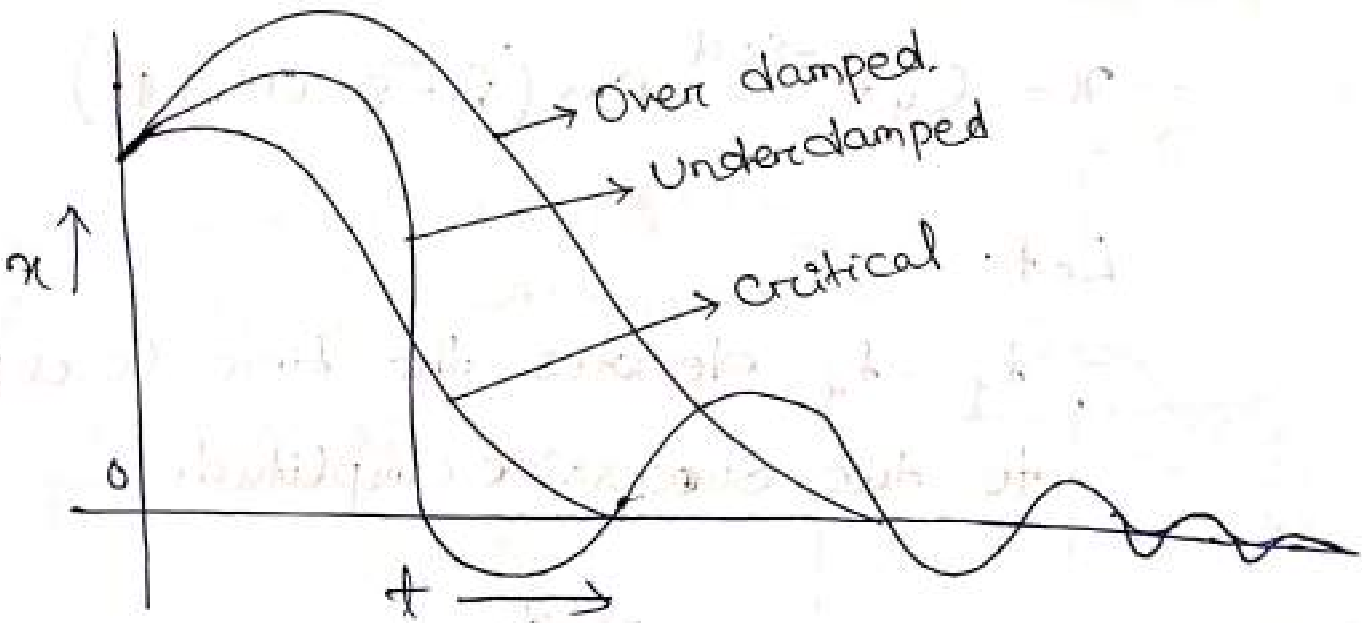
* $\boxed{\sqrt{1-\varepsilon^2} \cdot \omega = \omega_d}$ → frequency of damped vibration

$$\omega = 2\pi f$$

$$= \frac{2\pi}{t}$$

$$\Rightarrow t = \frac{2\pi}{\omega}$$

$$\boxed{t_d = \frac{2\pi}{\omega_d}}$$



$$x = C_3 e^{-\varepsilon\omega t}\left(\sin\sqrt{1-\varepsilon^2}\,\omega t + \phi_1\right)$$

$$= C_4 e^{-\varepsilon\omega t}\left(\cos\sqrt{1-\varepsilon^2}\,\omega t + \phi_2\right)$$

ϕ = phase difference.

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Logarithmic Decrement (σ)

Dt- 18/02/2020

→ It is defined as the natural logarithm of the ratio of any two successive amplitude on the same side of the mean line.

($\log_e = \ln$)

$$\boxed{\sigma = \ln \frac{x_1}{x_2}}$$

$$x = C_4 e^{-\varepsilon \omega t} \cos\left(\sqrt{1-\varepsilon^2}\,\omega t + \phi_2\right)$$

Let

t_1, t_2 denotes the time corresponding to two successive amplitude.

$$\frac{x_1}{x_2} = \frac{\cancel{C_4} e^{-\varepsilon \omega t_1} \cos\left(\sqrt{1-\varepsilon^2}\,\omega t_1 + \phi_2\right)}{\cancel{C_4} e^{-\varepsilon \omega t_2} \cos\left(\sqrt{1-\varepsilon^2}\,\omega t_2 + \phi_2\right)}$$

$$= e^{-\varepsilon\omega t_1 - (-\varepsilon \omega t_2)} \times \frac{\cos\left(\sqrt{1-\varepsilon^2}\,\omega t_1 + \phi_2\right)}{\cos\left(\sqrt{1-\varepsilon^2}\,\omega t_2 + \phi_2\right)}$$

$$= e^{-\varepsilon\omega(t_1 - t_2)} \times \frac{\cos\left(\sqrt{1-\varepsilon^2}\,\omega t_1 + \phi_2\right)}{\cos\left(\sqrt{1-\varepsilon^2}\,\omega t_2 + \phi_2\right)}$$

Let,

$$t_2 = t_1 + t_d$$

$$t_d = \frac{2\pi}{\omega_d}$$

$$\boxed{\frac{e^x}{e^y} = e^{x-y} \qquad e^x \times e^y = e^{x+y}}$$

t_d = Time period of damped Vibration.

Now the term $e^{-\varepsilon\omega(t_1 - t_2)} \times \dfrac{\cos(\sqrt{1-\varepsilon^2}\,\omega t_1 + \phi_2)}{\cos(\sqrt{1-\varepsilon^2}\,\omega t_2 + \phi_2)}$

The term

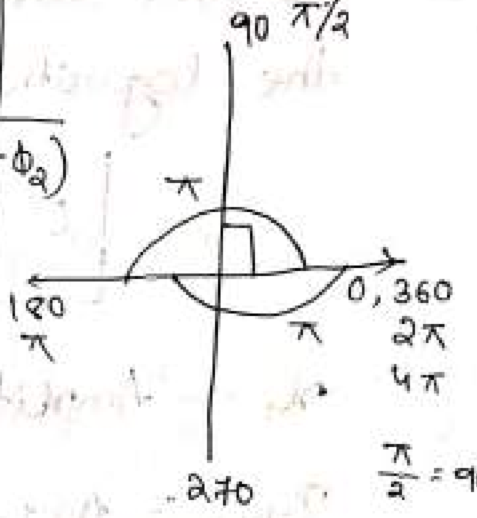
$$\frac{\cos(\omega_d t_1 + \phi_2)}{\cos(\omega_d t_2 + \phi_2)} = \frac{\cos(\omega_d t_1 + \phi_2)}{\cos(\omega_d (t_1 + t_d) + \phi_2)}$$

$$= \frac{\cos(\omega_d t_1 + \phi_2)}{\cos\left[\omega_d \left(t_1 + \frac{2\pi}{\omega_d}\right) + \phi_2\right]}$$

$$= \frac{\cos(\omega_d t_1 + \phi_2)}{\cos\left(\omega_d t_1 + \omega_d \frac{2\pi}{\omega_d} + \phi_2\right)}$$

$$= \frac{\cos(\omega_d t_1 + \phi_2)}{\cos\left(\omega_d t_1 + 2\pi + \frac{\theta}{2}\right)}$$

$$= \frac{\cos(\omega_d t_1 + \phi_2)}{\cos\left[(\omega_d t_1 + \phi_2) + 2\pi\right]} = 1$$



$\cos\theta = \cos(0 + \theta) = x$

$\cos(2\pi + \theta) = x$

$\dfrac{\cos\theta}{\cos(2\pi + \theta)} = x$

Now,

$$\frac{x_1}{x_2} = e^{-\varepsilon\omega(t_1-t_2)} = e^{-\varepsilon\omega(t_1-t_1+t_d)} = e^{\varepsilon\omega t_d}$$

$$\Rightarrow \frac{x_1}{x_2} = e^{\varepsilon\omega\frac{2\pi}{\omega_d}} = e^{\frac{\varepsilon\omega\cdot 2\pi}{\sqrt{1-\varepsilon^2}\,\omega}} = e^{\frac{2\pi\varepsilon}{\sqrt{1-\varepsilon^2}}}$$

$$\Rightarrow \log_e \frac{x_1}{x_2} = \log_e e^{\frac{2\pi\varepsilon}{\sqrt{1-\varepsilon^2}}}$$

$$\Rightarrow \boxed{\sigma = \frac{2\pi\varepsilon}{\sqrt{1-\varepsilon^2}} = \log_e \frac{x_1}{x_2} = \ln \frac{x_1}{x_2}}.$$

$\Rightarrow$ As ε is very small, $\boxed{\sigma \simeq 2\pi\varepsilon}$. $(\log_e e^K = K)$

If the system exicute 'n' number of cycle, the logarithm is decrement

$$\boxed{\sigma = \frac{1}{n}\log_e \frac{x_1}{x_{n+1}}}.$$

x_1 = Amplitude at position

x_{n+1} = Amplitude after 'n' cycles.

Dt - 28/02/2020

Proof

$$\sigma = \ln \frac{x_1}{x_2} = \log_e \frac{x_1}{x_2}$$

$$\Rightarrow \frac{x_1}{x_2} = e^{\sigma}$$

$$\frac{x_2}{x_3} = e^{\sigma}$$

$$\frac{x_3}{x_4} = e^{\sigma}$$

$$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \cdots\cdots \frac{x_n}{x_{n+1}} = e^{\sigma}$$

$k = \log_e y$
$\Rightarrow y = e^k$

$\log_e e^k = k$

$y = \ln k$
$\Rightarrow k = e^y$

$$\Rightarrow \frac{x_1}{x_2} \times \frac{x_2}{x_3} \times \frac{x_3}{x_4} \times \cdots\cdots \times \frac{x_n}{x_{n+1}} = (e^{\sigma})^n = e^{n\sigma}$$

$$\Rightarrow \frac{x_1}{x_{n+1}} = e^{n\sigma}$$

Applying $\log_e$ both sides, we get.

$$\Rightarrow \log_e \frac{x_1}{x_{n+1}} = \log_e e^{n\sigma}$$

$$\Rightarrow \log_e \frac{x_1}{x_{n+1}} = n\sigma \Rightarrow \boxed{\sigma = \frac{1}{n} \log_e \left(\frac{x_1}{x_{n+1}}\right)}.$$

$\frac{x_1}{x_2} \times \frac{x_2}{x_3} = e^{\sigma} \times e^{\sigma} = (e^{\sigma})^2$
$= e^{2\sigma}$

$\frac{x_1}{x_2} \times \frac{x_2}{x_3} \times \frac{x_3}{x_4} = e^{\sigma} \times e^{\sigma} \times e^{\sigma}$
$= (e^{\sigma})^3 = e^{3\sigma}$

Vibrational Energy and logarithimic decreament :-

! - factorial

Let, x_1 and x_2 are two successive amplitude.

Expansion !-

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} \cdots$$

$$\sigma = \ln \frac{x_1}{x_2}$$

$$\Rightarrow \frac{x_1}{x_2} = e^{\sigma}$$

$$\Rightarrow \frac{x_2}{x_1} = \frac{1}{e^{\sigma}} = e^{-\sigma}$$

Now expanding $e^{-\sigma}$,

$$e^{-\sigma} = 1 - \sigma + \frac{\sigma^2}{2!} - \frac{\sigma^3}{3!} + \cdots$$

Let

E_1 = Is the vibrational Energy at amplitude x_1.

E_2 = Is the vibrational Energy at amplitude x_2.

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$$E_1 = \frac{1}{2} k x_1^2$$

$$E_2 = \frac{1}{2} k x_2^2$$

Now,

$$\frac{E_1 - E_2}{E_1} = \frac{E_1}{E_1} - \frac{E_2}{E_1}$$

$$\boxed{\frac{E_2}{E_1} = \frac{\cancel{\frac{1}{2}}\cancel{k} x_2^2}{\cancel{\frac{1}{2}}\cancel{k} x_1^2} = \frac{x_2^2}{x_1^2} = \left(\frac{x_2}{x_1}\right)^2}$$

$$\Rightarrow \frac{\Delta E}{E_1} = 1 - \frac{E_2}{E_1}$$

$$\Rightarrow \frac{\Delta E}{E_1} = 1 - \left(\frac{x_2}{x_1}\right)^2$$

$$= 1 - \left(e^{-\sigma}\right)^2$$

$$= 1 - e^{-2\sigma}$$

$$= 1 - \left[1 - 2\sigma + \frac{(2\sigma)^2}{2!} - \frac{(2\sigma)^3}{3!} + \frac{(2\sigma)^4}{4!} \cdots\right]$$

$$= \cancel{1} - \cancel{1} + 2\sigma - \frac{(2\sigma)^2}{2!} + \frac{(2\sigma)^3}{3!} - \frac{(2\sigma)^4}{4!} + \cdots$$

$$\Rightarrow \frac{\Delta E}{E_1} = 2\sigma - \frac{(2\sigma)^2}{2!} + \frac{(2\sigma)^3}{3!} - \cdots$$

for small value of σ, higher power of 'σ' can be neglected.

$$\Rightarrow \frac{\Delta E}{E_1} = 2\sigma$$

$$\Rightarrow \sigma = \frac{\Delta E}{2E_1} \Rightarrow \boxed{\sigma = \frac{E_1 - E_2}{2E_1}}$$

$\Delta E = E_1 - E_2$ = Energy dicipeted in one cycle.

Formula Remember :-

* Free damping vibration

Viscous Damping

$F = \frac{\mu A}{t} \dot{x}$

A = Area
t = thickness
F = Force.

$\Rightarrow F = c\dot{x}$

$c = \frac{\mu A}{t}$ = viscous damping co-efficient

$$\boxed{m\ddot{x} + c\dot{x} + kx = 0}$$

* Energy dissipation (ΔE)

$$\boxed{\Delta E = \pi c \omega A^2}$$

maximum kinetic energy $E = \frac{1}{2} m \omega^2 A^2$

specific damping ratio $\beta = \frac{\Delta E}{E}$

$$\boxed{\beta = \frac{2\pi c}{m\omega}}$$

* Coulomb damping

$$\omega_n = \sqrt{\frac{k}{m}}$$

* Differential eq^n of damped free vibration :-

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$x = A_1 e^{u_1 t} + A_2 e^{u_2 t}$$

$$u_1, u_2 = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

A_1, A_2 = Arbitary Constant.

→ Critical damping Constant (C_c)

$$C_c = 2m\sqrt{\frac{k}{m}} = 2m\omega = 2\sqrt{km}$$

→ Damping Ratio $\boxed{\varepsilon = \frac{c}{C_c}} = \frac{c}{2\sqrt{km}}$

* $\frac{c}{2m} = \frac{c}{C_c} \times \frac{C_c}{2m} = \varepsilon\omega$

$$\therefore u_1, u_2 = -\varepsilon\omega \pm \sqrt{(\varepsilon\omega)^2 - \omega^2}$$

$$= \left[-\varepsilon \pm \sqrt{\varepsilon^2 - 1}\right]\omega$$

(i) Critical Damping ($\varepsilon = 1$)

$$x = (A_1 + A_2 t)e^{-\omega t}$$

(ii) Under damping $(\varepsilon < 1)$

$$x = A_1 e^{-\varepsilon \omega t} \sin\left(\sqrt{1-\varepsilon^2}\, \omega t + \phi\right)$$

(iii) Over damping $(\varepsilon > 1)$

$$x = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t}$$

* Time period

$$\rightarrow T = \frac{2\pi}{\omega}, \quad f = \frac{1}{T} = \frac{\omega}{2\pi}$$

* Logarithmic Decrement (σ)

$$\sigma = \ln \frac{x_1}{x_2}$$

x_1, x_2 = two succesive amplitudes.

* $\sqrt{1-\varepsilon^2}\, \omega = \omega d$ (frequency of damped vibration)

$$\sigma = \log_e \frac{x_1}{x_2} = \frac{2\pi\varepsilon}{\sqrt{1-\varepsilon^2}} = \frac{1}{n} \ln \frac{x_1}{x_{n+1}}$$

$$\sigma = \frac{\Delta E}{E_1} = \frac{E_1 - E_2}{E_1} \qquad \boxed{\sigma = 2\pi\varepsilon}$$

$$E_1 = \frac{1}{2} k x_1^2, \quad E_2 = \frac{1}{2} k x_2^2$$

ΔE = Energy dissipated in one cycle.

$$\boxed{\frac{1}{n-1} \log_e \frac{x_1}{x_n} = \frac{2\pi\varepsilon}{\sqrt{1-\varepsilon^2}}}$$

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